RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, MARCH 2021

FIRST YEAR [BATCH 2020-23]

Date : 26/03/2021

PHYSICS [HONOURS]

Time : 11 am - 1 pm

Paper: II [CC 2]

Full Marks : 50

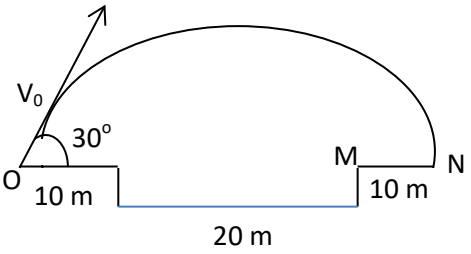
Answer **any five** questions: [5 × 10]

1. a) What do you mean by Galilean invariance?

   b) A rocket ascends from rest in a uniform gravitational field by objecting exhaust with constant speed u. Assume that the rate at which man is expelled is given by $dm/dt = \gamma m$, where m is the instantaneous man of the rocket and γ is a constant, and that the rocket is retarded by air resistance with a force $mb\nu$, where b is a constant. Find the velocity of the rocket as a function of time. [3+7]

2. a) A projectile is to be launched at an angle of 30^o so that it falls beyond the pond of length 20 meters as shown in the figure.

   i) What is the range of values of the initial velocity so that the projectile falls between points Mat.



   ii) Find the Rocket equation.

   b) Find the Rocket equation. [5+5]

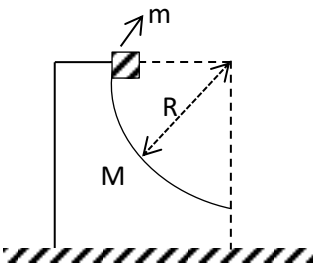
3. a) If potential energy, $u = 3x - x^3$

   Find the position of equilibrium and type of equilibrium.

   b) Define conservative force and non-conservative force.

   c) What is the advantages of centre of mass frame than laboratory frame?

   d)



A small cube of mass m slides down a circular path of radius R cut into a large block of mass M as shown. M rests on a table and both blocks move without friction. The blocks are initially rest and m starts from the top of the path.

Find the velocity ν of the cube as it leaves the block.

[(2+2)+2+2+2]

4. a) Show that if the total linear momentum of a system of particles is zero, the angular momentum of the system is the same about all origins.

   b) Find angular momentum of a particle of mass 'm' moving with speed 'v' with respect to origin

   i) along x axis

   ii) along y axis

iii) along x axis when y = b and with respect to point 'p'. Where p is any point on axis x when particle is moving along y axis.

c) Determine whether the force is conservative, and the potential energy function.

$\vec{F} = xyz\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)$ [3+4+3]

5. a) A homogeneous and isotropic cube is under stress along three mutually perpendicular directions *OX*, *OY* and *OZ*. Deduce the condition under which the linear strains along *OX* and *OY* will be equal while that along *OZ* will be zero.

b) Show that for the same cross-sectional areas, the beam of a square cross-section is stiffer than one of circular cross-section of the same length and material. Show also that for a given load, the depression in two cases are in the ratio $3:\pi$.

c) Liquid of density ρ and coefficient of viscosity η flows in stream-line through a capillary tube of radius R and length l. The velocity of the liquid at a distance r from the axis of the tube is given by $v(r) = v_0(1 - r^2/R^2)$. Find (i) the kinetic energy of the liquid within the volume of the tube, and (ii) the pressure difference between the ends of the tube. [3+3+(2+2)]

6. a) Derive an expression for the gravitational field inside a sphere of radius R when the mass density at a point is $\rho = a + br^2$, where r is the distance of the point from the centre of the sphere.

b) Suppose a small spherical planet has a radius of 10 km and a mean density of 5 g/cm^3.

(i) What would be the acceleration due to gravity at its surface?

(ii) What would a man weight on this planet if he weighted 80 kg on Earth?

c) At apogee of 300 km from the Earth's surface, two space ships in the same elliptical path are 150 m apart. How far apart will they be at perigee 250 km assuming that they drift without altering their path in anyway?

d) A particle moving in a central force field travels in a path which is the cycloid given by,

$r = a(1 - \cos\theta)$. Find the law of force.

e) A particle of mass m moves in a central force field defined by $\vec{F} = -k\hat{r}/r^3$. Prove that if E is the total energy supplied to the particle then its speed is given by $v = \sqrt{\frac{k}{mr^2} + \frac{2E}{m}}$. [2+2+2+2+2]

7. a) An xyz coordinate system rotates about the z-axis with angular velocity $\vec{\omega} = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath}$ with respect to a fixed XYZ coordinate system where t in the time. The origin of the xyz system has position vector $\vec{R} = t\hat{\imath} - \hat{\jmath} + t^2\hat{k}$ with respect to the XYZ system. If the position vector of a particle is given by $\vec{r} = (3t + 1)\hat{\imath} - 2t\hat{\jmath} + 5\hat{k}$ relative to the moving system, then find the (i) apparent velocity, (ii) true velocity, (iii) apparent acceleration, (iv) Coriolis acceleration (v) centripetal acceleration and (vi) true acceleration.

b) If a projectile is fired with initial velocity $v_1\hat{\imath} - v_2\hat{\jmath} + v_3\hat{k}$ from the origin of a coordinate system fixed relative to the Earth's surface at colatitude λ, prove that its position at any later time t will be given by

$x = v_1 t + \omega v_2 t^2 \cos\lambda$,

$y = v_2 t - \omega t^2(v_1 \cos\lambda + v_2 \sin\lambda) + \frac{1}{3}\omega g t^3 \sin\lambda$,

$z = v_3 t + \omega v_2 t^2 \sin\lambda + \frac{1}{2} g t^2$,

neglecting the terms involving ω^2. [(1+1+1+1+1+1)+4]

8. a) A particle vibrates with simple harmonic motion of amplitude 5 cm and a period of 6 s. How long does it take to move from one end of its path to a position 2.5 cm from the equilibrium position on the same side?

b) Consider a pendulum of length of length l and a bob of mass m at its end moving through oil with θ decreasing. The massive bob undergoes small oscillations, but the oil retards the bob's motion with a resistive force proportional to the speed with $F_{res} = 2m\sqrt{\frac{g}{l}}(l\dot{\theta})$. The bob is initially pulled back at $t = 0$ with $\theta = \alpha$ and $\dot{\theta} = 0$. Find the angular displacement θ and velocity $\dot{\theta}$ as a function of time.

c) The amplitude of a weakly damped oscillator decrease to 20% of the initial value after five consecutive cycles of oscillations. Determine the damping coefficient p (viscous force $-pv$) if $k = 40$ Nm^{-1} and $m = 2.5$ kg.

d) An object of mass 0.1 kg is hung from a spring whose spring constant is 100 Nm-1. A resistive force $-pv$ acts on the object, where v is the velocity and $p = 1$ Nsm-1. The object is subjected to a harmonic driving force $F = F_0 \cos \omega t$ where $F_0 = 2$N and $\omega = 50$ rads-1. In the steady state what is the amplitude of oscillations and the phase relative to that of the applied force? [2+3+2+3]

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